Lecture 1

Introduction: Graphs and Probability

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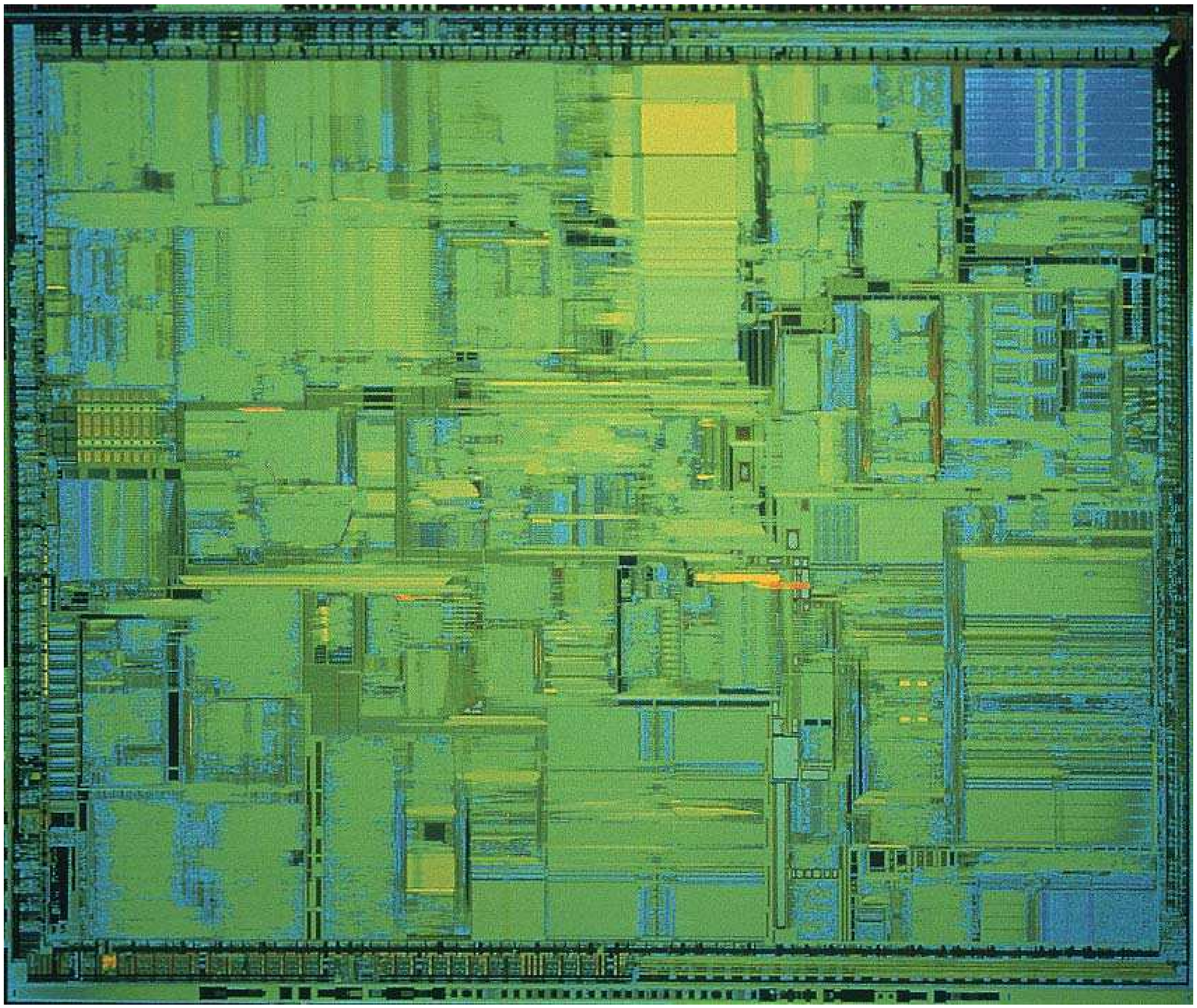
Overview

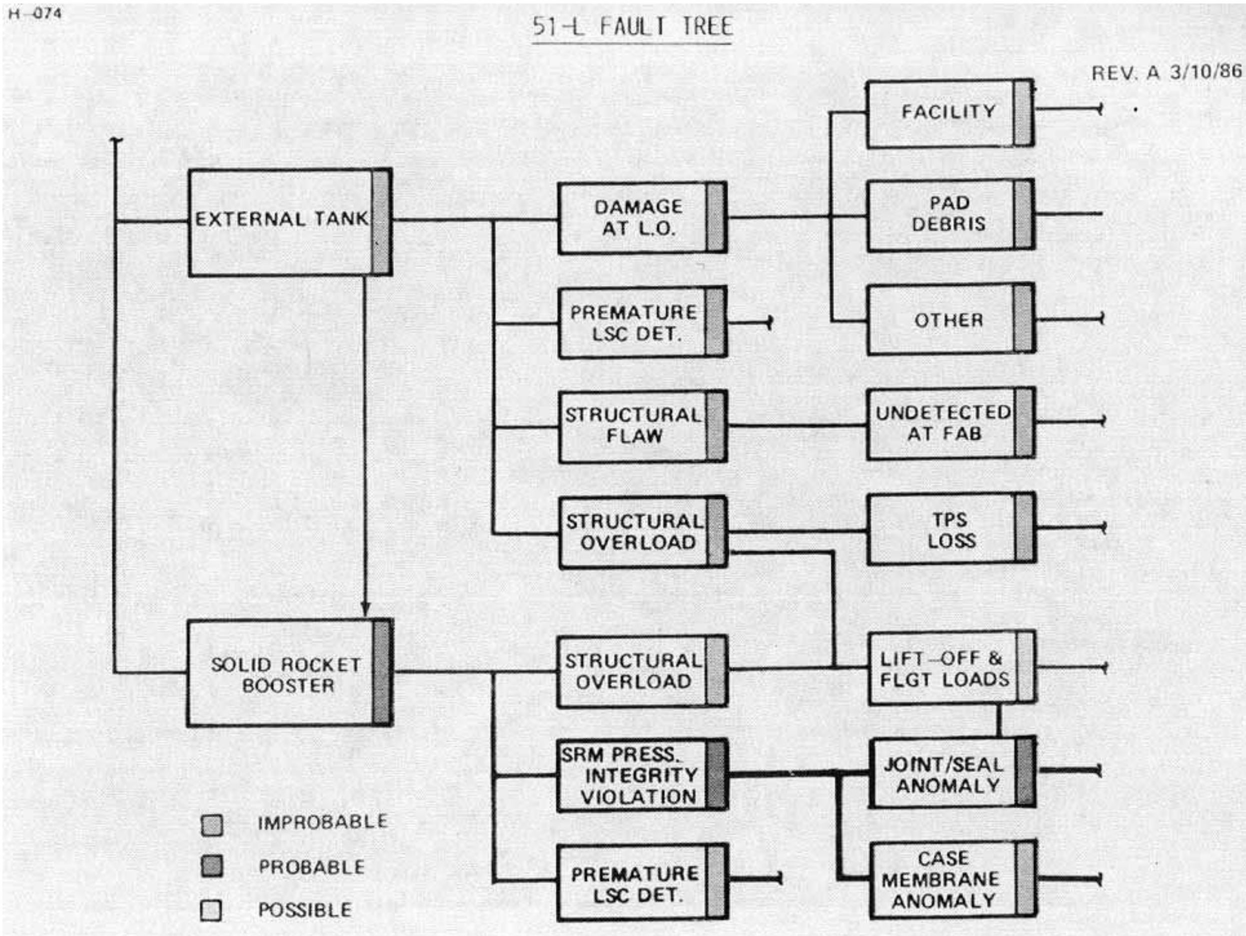
- **Graphical Models**
- Probabilities
- Modelling
- Modelling with Probabilistic Graphical Models

A Very Big Graphical Model



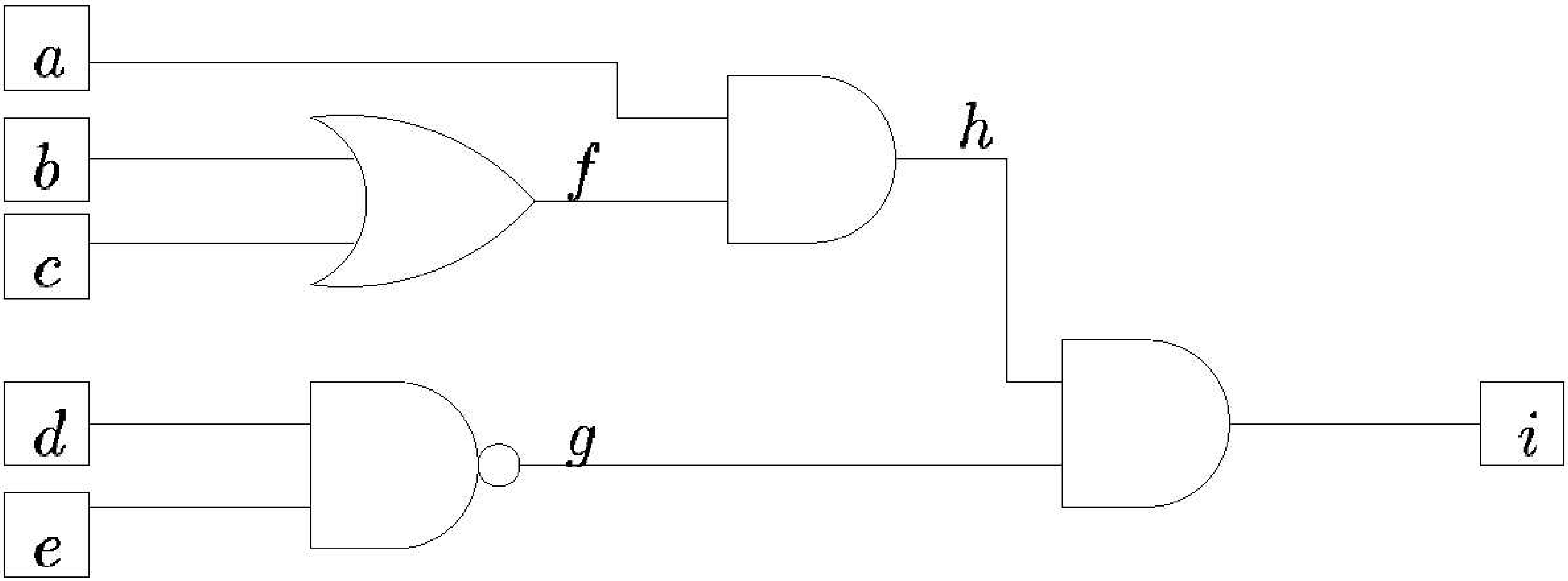
"Pentium 3" oil painting by Silicon Valley's premier corporate artist Jens Gebhart.

An Infamous Graphical Model



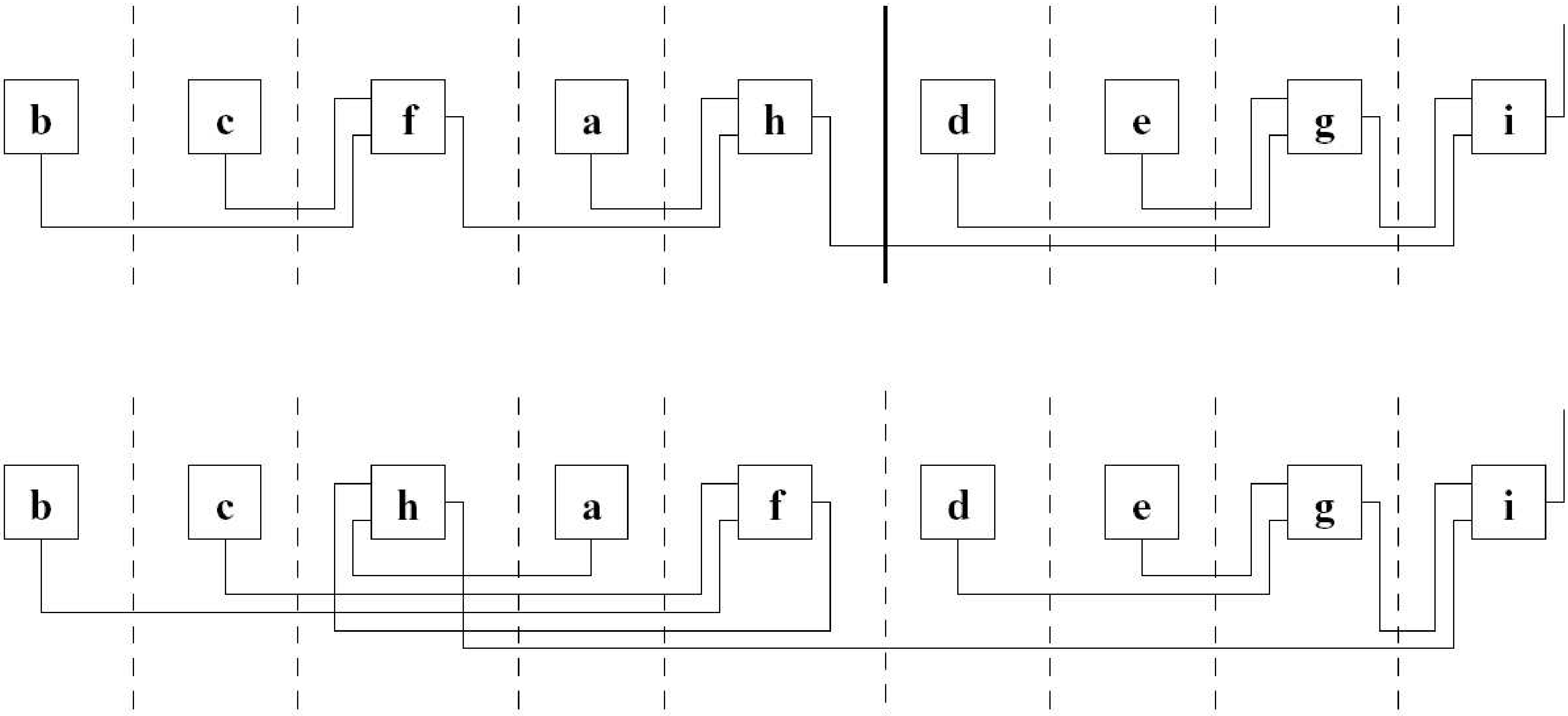
Part of a **fault tree** from Report of the PRESIDENTIAL COMMISSION on the Space Shuttle Challenger Accident, 1986.

Constraint graphs from Circuits



$$(\bar{b}+f)(\bar{c}+f)(b+c+\bar{f})(d+g)(e+g)(\bar{d}+\bar{e}+\bar{g})(a+\bar{h})$$
$$(f+\bar{h})(\bar{a}+\bar{f}+h)(h+\bar{i})(g+\bar{i})(\bar{h}+\bar{g}+i)$$

Constraint Graphs from Circuits, linear layout



Two linear layouts.

A Microsoft Perspective

Gates responded . . . Microsoft's competitive advantage . . . was its expertise in 'Bayesian networks.' . . . Is Gates onto something? Ask any other software executive about anything 'Bayesian' and you're liable to get a blank stare. Is this alien-sounding technology Microsoft's new secret weapon?

`Los Angeles Times, October 28, 1996`

Graphical Models

- ubiquitous in Computer Science
- directed, undirected, and hypergraphs graphs used;
- some applied examples:
  - computer networks (PCs, printers, hubs, . . .)
  - telephone circuits, the Internet, . . .
  - unified modelling language, entity relationship diagram, . . .
  - circuit-level design for CPU chip
- some theoretical examples:
  - traveling salesman problem (TSP)
  - graph coloring
  - constraint graphs (e.g., for satisfiability)
  - hypergraph partitioning
- what is the *most studied* and *most experimented with*?

Graphical Models, cont.

- some model exact processes
- some model approximate processes:
  - expert systems' models
  - fault trees
  - models in image, speech, vision, etc.
- represent the structural aspects of a problem
- pictorial representations not necessary, just useful
- graphs focus on properties of binary relations; hypergraphs focus on n-ary relations
- we will cover some aspects of general theory useful for probabilistic graphs

Graphical Models: Basic Theory

The following definitions are given on MathWorld (`http://mathworld.wolfram.com`): graph, directed acyclic graph, forest, tree, degree, outdegree and indegree, clique, induced subgraph, and connected component.

Other definitions we will introduce:

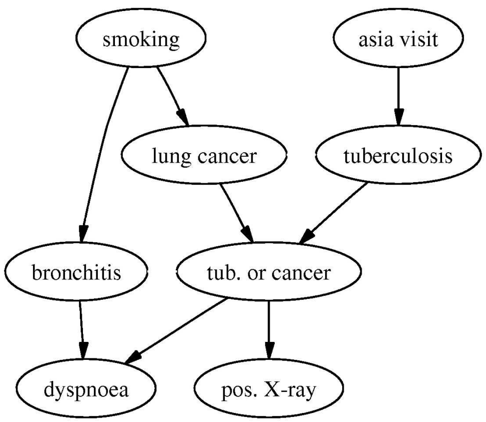
- parents, ancestors, ancestral set
- children, descendants, descendant set
- cut-width (of elimination ordering)

Ancestors and Descendants

- Graph is a pair of sets (V, E), for V the vertex set and E the edge set (a subset of all possible pairs from V).
- If $(x, y) \in E$, then x is a *parent* of y and y a *child* of x.
- The *transitive closure* of relation R is given by $R^* = I \cup R \cup R^2 \cup R^3 \cup \ldots$.

$$
\begin{aligned}
\text{ancestors}(S) &\equiv \text{parents}^*(S) - S \\
\text{descendants}(S) &\equiv \text{children}^*(S) - S \\
S \text{ is an } \textit{ancestral set} &\equiv \text{ancestors}(S) = \emptyset \\
S \text{ is a } \textit{descendant set} &\equiv \text{descendants}(S) = \emptyset
\end{aligned}
$$

Ancestors and Descendants, examples



$$
\begin{aligned}
\text{ancestral sets} &:: \{A\},\ \{S\},\ \{A,T\},\{S,L,B\},\\
&\quad\ \{A,S,T,L,B\},\ldots\\
\text{descendant sets} &:: \{X\},\{D\},\{X,E,D\},\\
&\quad\ \{T,L,X,E,D\},\ldots\\
\text{ancestors}(X) &= \{A,T,E,L,S\}\\
\text{descendants}(A) &= \{T,E,X,D\}
\end{aligned}
$$

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Probability

Consider the probability of:

1. A strong virus alert will be announced for Windows in the next week.
2. The Euro will go above US$1.30 later in 2005.
3. The height of a Finnish male is 180–185cm.
4. A Finnish male is tall.

- Probabilities must be for well-defined events: (4) is not, (1) possibly not.
- Those for one-off unrepeatable events (1), (2) cannot be sampled.
- Probabilities are always context dependent: (2) will vary during 2005, (3) changes when in a päiväkoti.
- Continuous events need to be discretized, like (3).

Probability Operations

- Probability or probability density is a function over discrete or continuous variables in a domain.
- Operations and Concepts:

Conditioning	$p(X\|Y) = p(X,Y)/p(Y)$
Marginalization	$p(X) = \sum_Y p(X,Y)$
Factoring	$p(X,Y) = p(X)p(Y\|X)$
Independence	$X \perp\!\!\!\perp Y \,\|\, Z \equiv p(X\|Y,Z) = p(X\|Z)$ when $p(Y,Z) > 0$
Expectation	ExpectedValue$(U(X)) = \sum_X U(X)p(X)$
Maximization	Max$_d$ExpectedValue$(U(X,d))$

- Summation replaced by integration in continuous domains.
- Probability density is *always* a theoretical abstraction because the world and all measurements we take are discrete.

Continuous Domains as an Abstraction

"John's height is 60π cm, or 188.49555921538759430775860299677017 40 cm"

"In Indonesia, a person with tertiary education earns an average 82% more than one with secondary qualifications"

- In the real world, the only evidence we see is discrete, and the only valid statements we can make are about discrete events.
- Continuous domains are a useful *abstraction*.
- *Numerical integration* and *measure theory* are about making a discrete approximation to a continuous probability distribution.
- Probabilities for point events can be made arbitrarily small or large depending on change of variables.

Constraint Satisfaction as Probabilistic Reasoning

Turn a CSP into a probabilistic reasoning task as follows:

$C(X)$ is constraint	$\leftrightarrow$	joint X table zero iff $\neg C(X)$
set X satisfies constraints	$\leftrightarrow$	$p(X) > 0$
set X fails constraints	$\leftrightarrow$	$p(X) = 0$
variable assignment	$\leftrightarrow$	conditioning
variable elimination	$\leftrightarrow$	marginalization
problem decomposition	$\leftrightarrow$	independence
local search	$\leftrightarrow$	Gibbs sampling

Similar mappings apply to optimization. Thus many relationships exist between graph algorithms for constrain satisfaction, optimization and probabilistic reasoning.

Models of Graph Coloring

Have a given graph whose N nodes on discrete variables $\mathscr{X}$ taking on 3 values representing colors. Let the constraint set $C(\mathscr{X})$ represent the set of constraints for the problem, e.g., if $X,Y \in \mathscr{X}$ connected in graph, then $(X \neq Y) \in C(\mathscr{X})$. Make the distributions for nodes be independent and uniform.

- number of solutions $3^N p(C(\mathscr{X}))$
- locally, each arc has a 1/3 chance of violation
- suppose average connectivity is C, gross independence approximation says $p(C(\mathscr{X})) = (2/3)^{CN/2}$.
- number of solutions $\approx 3^N (2/3)^{CN/2}$

Models of Graph Coloring, cont.

Number of solutions $\approx 3^N (2/3)^{CN/2}$ for C the average connectivity.

- for $C \ll 5.3$ solutions $\ll 1$, i.e., few solutions even in small subgraphs so backtrack quickly exhausts space and proves no solution
- for $C \gg 5.3$ solutions $\gg 1$, i.e., many solutions, even for subgraphs, so solutions on subgraphs often become full solutions, so local search quickly finds one
- for $C \approx 5.3$ solutions ≈ 1, graph coloring is hard!

Experimentally confirmed for $C = 5.4$ Cheeseman et al. (1995). Gory detail in Turner (1988).

Three Kinds of Probabilities for the Bayesian

Proportions: e.g., probability the height of a Finnish male is 140–145cm

- true proportions about the world
- measurable but sometimes approximated
- Kolmogorov Axioms hold

Beliefs: e.g., probability Euro will go above US$1.30 later in 2005

- subjective beliefs, particular on one-off events
- not practically measurable, realm of Decision Theory
- Cox's Axioms, Dutch books or other schemes justify

Beliefs about Proportions: e.g., probability density function on proportions for rolls of die for biased dice on Wray desk

- subjective beliefs about true proportions
- not practically measurable (Wray wont let you into the room for long enough)
- realm of Bayesian Statistics

Assumptions About Probability as Belief

Standard frameworks for justifying the use of probabilities to model beliefs (i.e., Bayesian reasoning) make the following (often hidden) **assumptions**:

- you have unlimited computational power
- you are a single agent seeking to optimize performance in directly measurable sense
- you are using a model family that includes a sufficiently close approximation to the "truth"

These become **dangerous** when:

- seeking to model "objective" reasoning
- using simple linear models for everything
- using an old CPU with low power and memory

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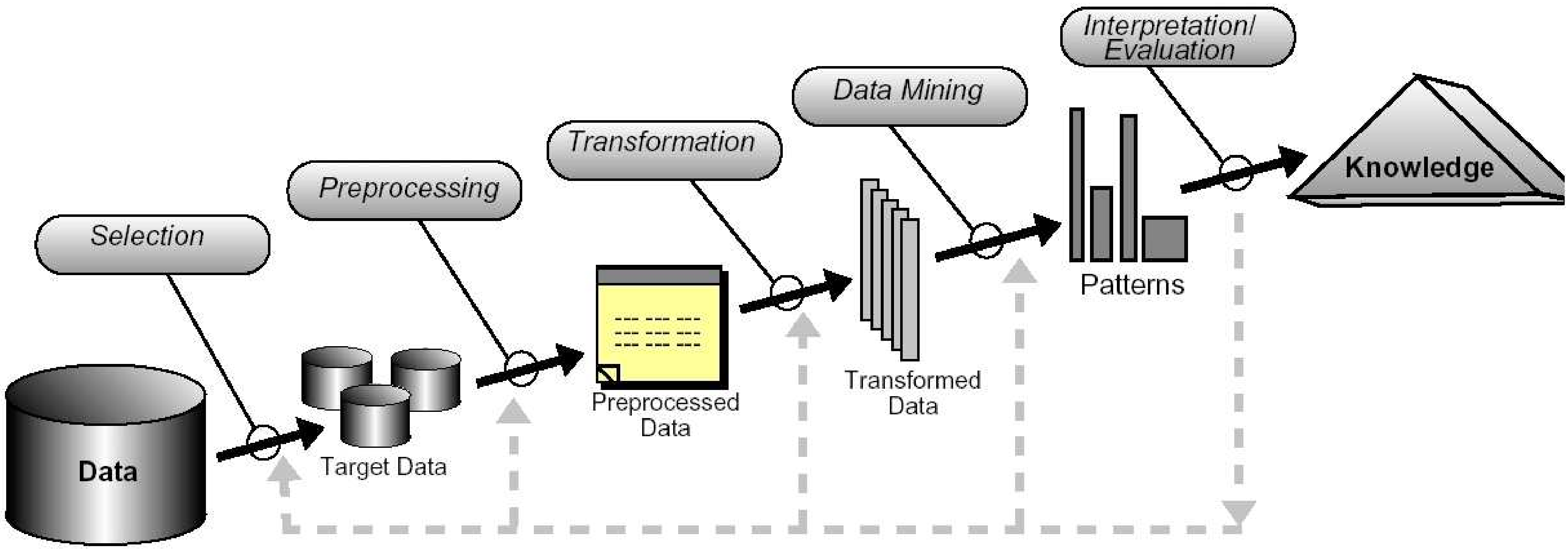
The Knowledge Discovery Process

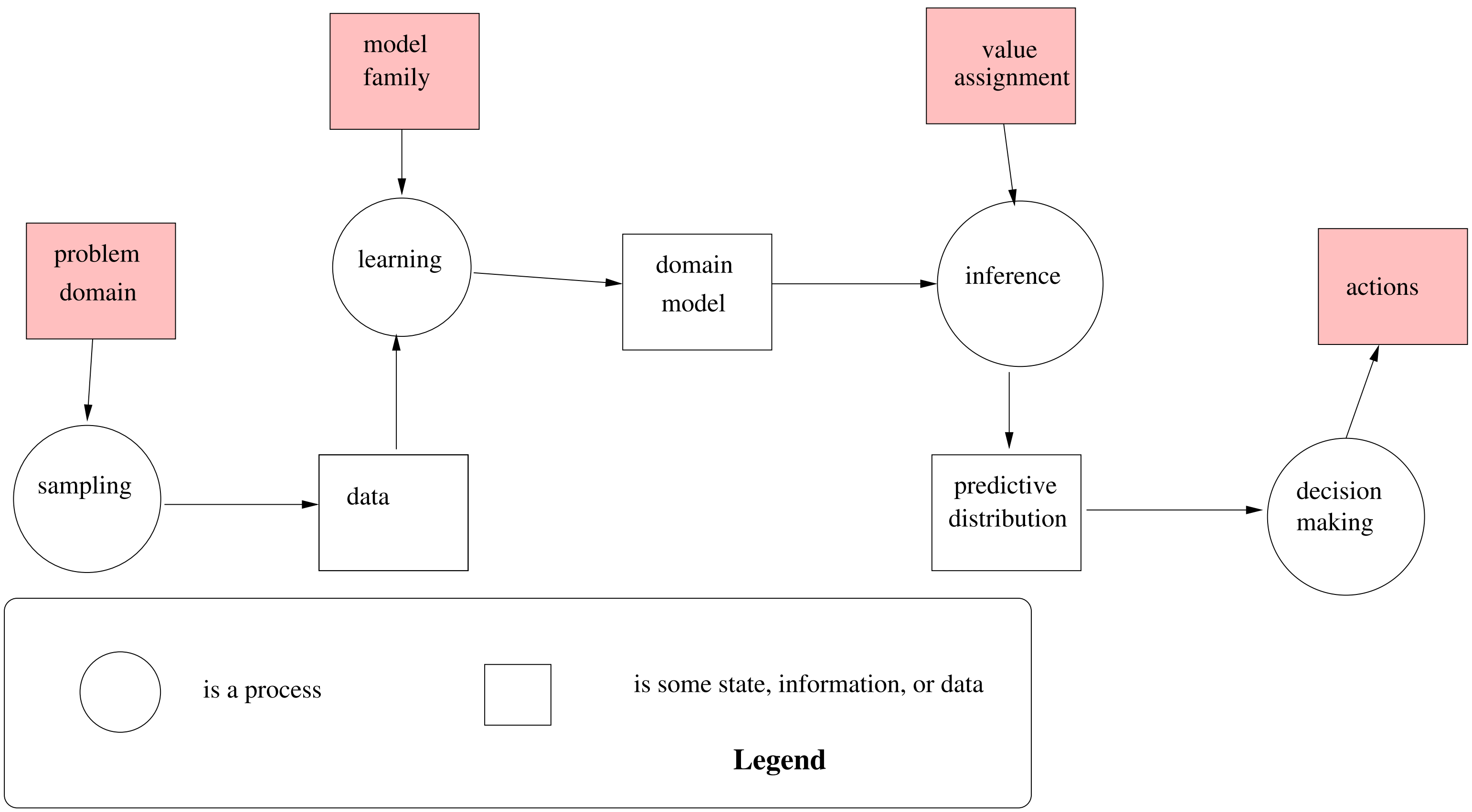
Is a process of iterative refinement. But represented here without a "model".



From Fayyad, Piatetsky-Shapiro & Smyth (1996)

The Modelling Process

Is a process of iterative refinement (though here static). A model represents a *generative* or *functional* model for the problem elements.



From Tirri's class notes for Three Concepts: Probability

Modelling Objectivity

Question: *The Royal Society wants to make a report about whether the new AIDS drug KylieKleptron cause dangerous side effects?*

- Their statement should not be subjective.
- Their statement should not have to be retracted in 12 months due to additional evidence.
- Their statement should reflect the considered opinion of a variety of different experts.

Objective reasoning using probabilities requires different arguments and methods beyond simple Bayesian probabilistic reasoning and is not covered here.

Modelling Dimensions: Tradeoffs/Art

- More data allows for better calibration of your model, but may be expensive or costly.
- Using more complex models might be closer to "truth", but might cause more computation.
- Human experts can often provide advice, but are fallible.
- Sometimes, just the choice of variables to use is the critical decision.
- Don't mess with the data! Model it.

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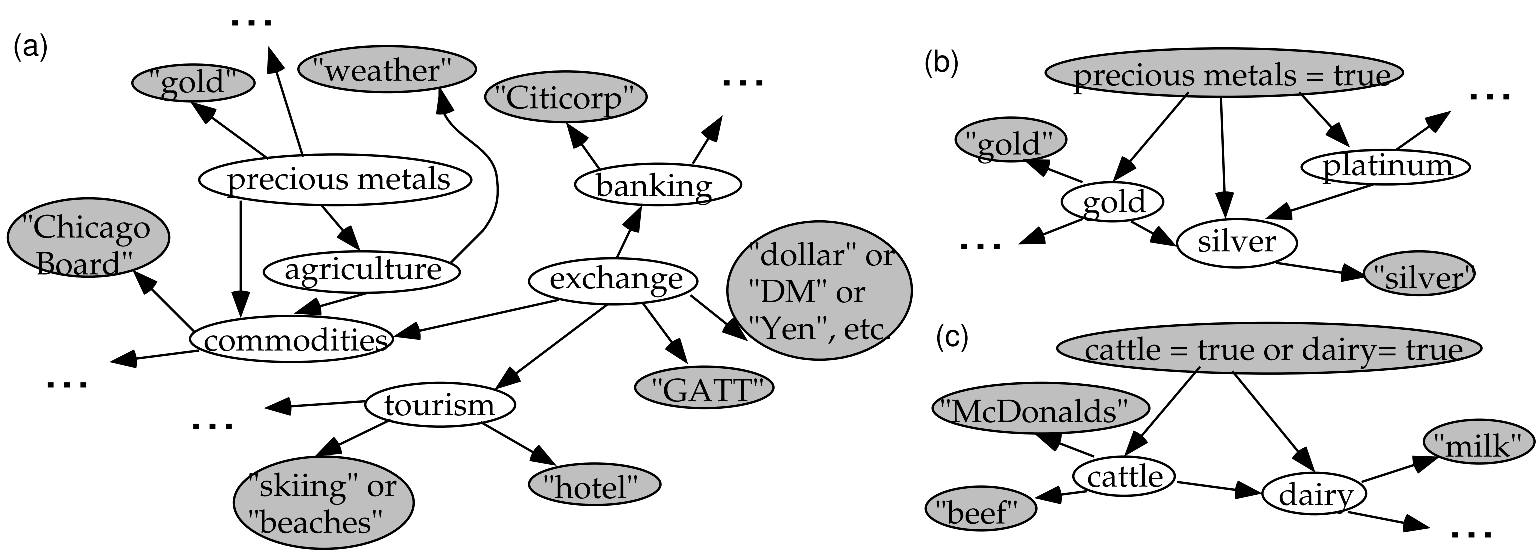
Probabilistic Graphical Models

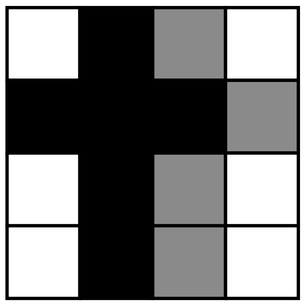
- different kinds of graphs for different kinds of information:
  - causes
  - dependencies, direct influences, . . .
  - actions, states, observables,. . .
- probabilistic relations are associated with graphs i.e., represents a set of probability equations
  - Gaussians, linear models
  - discrete distributions
  - neural nets
- standard graph operations (variable elimination, decomposition, etc.) apply

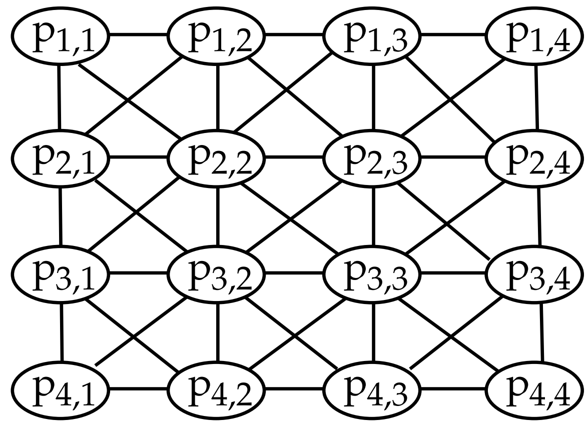
Topic-Subtopic Models for News



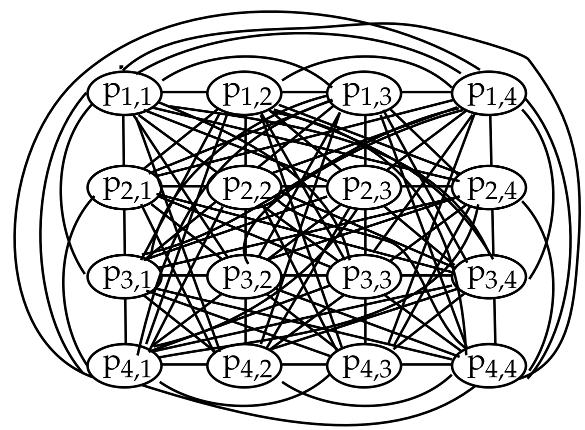
Three views ((a), (b), (c)) of a topic-subtopic model for Reuters news articles classified according to the standard topic level topics (commodities, cattle, etc.). Word features for articles are given in quotes, topics unquoted.

Simple Image Models





Simple 4×4 image. Top graph says all pixels influenced only by their neighbour's values. Bottom graph says everyone influences everyone else.



Next Week

- Review Shachter's notes on "Examples without Numbers"
- Review Scheines' online tutorial on "d-separation" (but don't spend too much time, we wont use this).
- Check out suggested project topics and consider a topic of your own choosing.