Nintendo Wii Fit -based balance testing to detect sleep deprivation: Approximate Bayesian computation -approach

Aino Tietäväinen, Lic. Phil.,
Prof. Edward Hæggström,
Prof. Jukka Corander
Dr. Michael Gutmann
Dr. Esko Keski-Vakkuri
Motivation


• Sleepiness causes accidents & deteriorates health.
  • 10-30% traffic accidents (yet, 60% drive while sleepy!)
  • Increases the risk for e.g. diabetes, depression, cardiovascular disease, cancer

• Compares to alcohol:
  Performance(17 h awake) = Performance(0.5% BAC)¹

• “Breathalyzer” for sleepiness?

Ultimate goal
- for the police
- for self-testing
Measuring balance


- Sleepiness affects CNS + CNS used in balancing = balance test for checking sleepiness?

- Easy and fast: 30s-60s of standing on the *Nintendo Wii Fit* Balance Board:
  - Portable
  - Wireless
  - Inexpensive
  - Valid & reliable¹
Measuring balance
Quantifying balance

- How to extract information?

1. Conventional
   - Related to sway amplitude, velocity and frequency

2. Nonlinear
   - “Algorithms”, quantify the structure of the sway signal
     - Fuzzy Sample Entropy (FSE, regularity of the signal)

3. Parameter inference from sway model
   - May be interpreted physiologically

Sway measures
Alert: 14 mm
Sleepy: 24 mm
Results

-Laboratory setting

- 15 subjects measured every hour for 24 hours
  → A reference curve
  - Challenge: circadian rhythm
  - Good correlation ($\rho=0.94$) between sleepiness model and FSE
  - “Most sleepy” detected with 75% accuracy
Parameters of interest:

- $P$, Active stiffness
- $D$, Active damping
- $\sigma$, Intensity of noise
- $\Delta$, Time delay
- $C_{ON}$, Level of control (related to $a_s$)

Parameter inference

- Approximate Bayesian Computation (ABC)

1Sisson et al., PNAS, 104(6), pp. 1760-1765, 2007

What we have:

- Likelihood function unavailable analytically in closed form
- ABC relies on calculating summary statistics $\Phi_{obs}$ and $\Phi_{sim}$
- How to choose?
- Sequential Monte Carlo ABC (SMC-ABC)

What we want:

- $P$, Active stiffness
- $D$, Active damping
- $\sigma$, Intensity of noise
- $\Delta$, Time delay
- $C_{ON}$, Level of control

ABC

- Measured COM
- Measured COP
- Simulated COM

Sway signal (mm)

Time (s)
In each iteration:

- Simulate with different candidate parameter values,
- Calculate $\Phi_{\text{obs}}$ and $\Phi_{\text{sim}}$ and the discrepancy $\rho$
- Continue until $N$ simulations produces $\rho \leq$ threshold $\varepsilon$
- When $N$ reached, lower $\varepsilon$ (new iteration). Sampling from the new proposal distribution.
- If maximum number of iterations done, stop.

Lintusaari et al., *Syst. Biol.*, (66)1, pp. e66-e82, 2017
Summary Statistics

- Amplitude -, velocity -, acceleration histograms & spectrum

- Discrepancy: \( \rho = \frac{1}{l} \sum_{i=1}^{l} \left| \frac{\Phi_{obs} - \Phi_{sim}}{\Phi_{obs} + \Phi_{sim}} \right| \),

where \( l=60 \) is the length of the summary statistics.
Results 1/5
-Simulated subjects

Inferred sway patterns represent the simulated ones.

- Summary statistics from measured COMs within 95% CIs of inferred summary statistics.
Results 2/5
-Simulated subjects

- Marginal posterior probability density functions for the five parameters of interest
Results 3/5
-Simulated subjects

- Estimated parameter values against true parameter values

![Graphs showing correlations between estimated and true values](image-url)
Problem with $D$?

Stochastic delay differential equation:

$$T_{tot} = T_g(t) - T_c(t) + T_d(t).$$

$$I \ddot{\theta}(t) = mgh\dot{\theta}(t) - \left[K\theta(t) + B\dot{\theta}(t) + f_P(\theta(t - \Delta)) + f_D(\dot{\theta}(t - \Delta))\right] + \sigma \xi(t)$$

\[
\begin{cases}
  f_P(\theta(t - \Delta)) = P\theta(t - \Delta) \\
  f_D(\dot{\theta}(t - \Delta)) = D\dot{\theta}(t - \Delta)
\end{cases}
\]

if \quad \theta(t - \Delta) \left(\dot{\theta}(t - \Delta) - a_s \theta(t - \Delta)\right) > 0, \text{ and } \theta^2(t - \Delta) + \dot{\theta}^2(t - \Delta) > r^2, \text{ and}

\[
\begin{cases}
  f_P(\theta(t - \Delta)) = 0 \\
  f_D(\dot{\theta}(t - \Delta)) = 0
\end{cases}
\]

otherwise.

$T_{tot} = T_g(t) - T_c(t) + T_d(t).$

$f_P$ 50-times larger than $f_D$
Results 4/5
- Real subjects

\[
\text{Inferred sway patterns represent the measured ones.}
\]

- Summary statistics from measured COMs mostly within the 95% CIs of inferred summary statistics.
Results 5/5
-Real subjects

- Marginal posterior probability density functions for the five parameters of interest
Sensitivity Analysis
-Simulated subjects
What next?


- SMC-ABC still slow (requires clusters)
  ⇒ Another ABC code: BOLFI¹ (Bayesian optimization in Likelihood-Free Inference)
- Some work with the summary statistics needed
- Sleepiness data (N=21, TA=24 h) needs to be implemented
Thank you!

Additional material: SMC-ABC

Sisson et al., PNAS, 104(6), pp. 1760-1765, 2007

**ABC-PRC Algorithm (corrected):**

PRC1 Initialize $\epsilon_1, \ldots, \epsilon_T$, and specify initial sampling distribution $\mu_1$.
Set population indicator $t = 1$.

PRC2 Set particle indicator $i = 1$.
PRC2.1 If $t = 1$ sample $\theta^{**} \sim \mu_1(\theta)$ independently from $\mu_1$.
If $t > 1$ sample $\theta^*$ from the previous population $\{\theta^{(i)}_{t-1}\}$ with weights $\{W^{(i)}_{t-1}\}$, and perturb the particle to $\theta^{**} \sim K_t(\theta | \theta^*)$ according to a transition kernel $K_t$.

Generate a data set $x^{**} \sim f(x | \theta^{**})$.
If $\rho(S(x^{**}), S(x_0)) \geq \epsilon$, then go to PRC2.1.

PRC2.2 Set

$$\theta^{(i)}_t = \theta^{**} \quad \text{and} \quad W^{(i)}_t = \begin{cases} \frac{\pi(\theta^{(i)}_t) / \mu_1(\theta^{(i)}_t)}{\mu_1(\theta^{(i)}_t)} & \text{if } t = 1 \\ \frac{\pi(\theta^{(i)}_t) / \sum_{j=1}^N W_{t-1}(\theta^{(j)}_{t-1}) K_t(\theta^{(i)}_t | \theta^{(j)}_{t-1})}{W_{t-1}(\theta^{(i)}_{t-1}) K_t(\theta^{(i)}_t | \theta^{(j)}_{t-1})} & \text{if } t > 1 \end{cases}$$

where $\pi(\theta)$ denotes the prior distribution for $\theta$. 
If $i < N$, increment $i = i + 1$ and go to PRC2.1.

PRC3 Normalize the weights so that $\sum_{i=1}^N W^{(i)}_t = 1$.
If $ESS = \left[ \sum_{i=1}^N (W^{(i)}_t)^2 \right]^{-1} < E$ then resample with replacement, the particles $\{\theta^{(i)}_t\}$ with weights $\{W^{(i)}_t\}$ to obtain a new population $\{\theta^{(i)}_t\}$, and set weights $\{W^{(i)}_t = 1/N\}$.

PRC4 If $t < T$, increment $t = t + 1$ and go to PRC2.